**al-FARABI KAZAKH NATIONAL UNIVERSITY**

**Faculty of Mathematics and Mechanics**

**Department of Mathematical and Computer Modelling**

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|  | **Approve**  **at a faculty academic council meeting**  **The transactions No , 2012**  **Dean of Faculty \_\_\_\_\_\_\_\_\_\_\_\_ D.Zh. Ahmed-Zaki** |

**SYLLABUS**

**Algebra and Analysis Numerical Methods**

**a second-year bachelor, specialty – Mathematics, autumnal term (second half-year), credit 3 (2-1-0)**

**Professor: Kanat Shakenov**

**Tel.: 3857478**

**e-mail:** [**shakenov@kaznu.kz**](mailto:shakenov@kaznu.kz)**, Kanat.Shakenov@kaznu.kz**

**room: 319**

**Trans-property: Algebra, Geometry, Mathematical Analysis, ODE, PDE, Functional Analysis, Computational Mathematics.**

**Post- property: Monte Carlo Methods, Mathematical Modelling, Computer Modelling, Numerical Mathematics, Numerical Fluid Mechanics.**

**Goals and objectives. I believe that modern computers can play a similar role in mathematics. This course presents the innovative approach that numerical methods should be considered as a practical laboratory for undergraduate mathematics courses. I think this is innovative because it is not the state of affairs we currently encounter. On the one hand, first-, second- and third-year students in mathematics, science, and engineering learn introductory mathematical concepts without making appropriate use of computer technology. On the other hand, upper-level courses on numerical methods put their emphases on specific topics such as computational algorithms, error analysis, convergence and stability, and coding and debugging procedures, with only passing references to the mathematical background, which is generally assumed to be known and understood beforehand.**

**The structure of the course.**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Weeks** | **Name of subject (theme)** | **duration** | **Students self-instruction (SSI) by subject** | |
| **Module 1. Numerical Methods of Linear Algebraic Equations Systems (NMLAES)** | | | | |
| 1  -  2  -  3  -  4 | **Lecture 1-8.** Systems of Linear Equations.Matrix Theory.Vector and Matrix Norms. Direct Methods: Gauss, Square Roots, Cholesky Factorization. Iterative Methods. General Schemes. The Sufficient Condition of convergence of the Iterative Process. Estimated Error. The Requisite and Sufficient Conditions of convergence of the Iterative Process. Gauss-Seidel method. Symmetrical Matrix. The Sufficient Condition of convergence of Gauss-Seidel methods. Minimal Residual (Discrepancy, Disparity) Methods and convergence. Method of Steepest Descent and convergence.  **Seminar 1-4.** Vector and Matrix Norms computing. LAES problem by Gauss, Square Roots, Cholesky Factorization, Iterative (iterative solution), Gauss-Seidel, Minimal Residual, Method of Steepest Descent methods solving. Computer Aided Management (CAM) system programming. | 8  4 | | SSI-1-4 Theory Matrix. Vector and Matrix Norms computing. |
| Module 2. General Iterative Methods of the Solutions Algebraic and Transcendental Equations | | | | |
| 5  -  6 | **Lecture 9-12.** General Iterative Methods of the Solutions Algebraic and Transcendental Equations. Compact Mapping Principle and their applications. Isolation of the Roots of Algebraic and Transcendental Equations. Intersecting (Chord) Methods, Newton (Tangent) Methods and Mixed methods. Newton – Kantorovich Methods. False Point Methods, Stephenson Methods, Wall Methods, Newton – Kantorovich Modifications Methods.  **Seminar 5-6.** Isolation of the Roots of Algebraic and Transcendental Equations.Problem solvingby Intersecting, Newton and Mixed, Newton – Kantorovich, False Point, Stephenson, Wall, Newton – Kantorovich Modifications methods. CAM system programming. | 4  2 | | **SSI-5-6**  Isolation of the Roots of Algebraic and Transcendental Equations. |
| **Module 3. Eigenvalues and Eigenvectors** | | | | |
| 7 | **Lecture 13-14.** Matrix Eigenvalue Problems. Iterative Methods (Numerical Methods). Properties of Eigenvalues. Power Iterations.  **Seminar 7.**  Problem solvingby Iterative methods. CAM system programming. | 2  1 | | **SSI-7**  Global Matrix Eigenvalue Problems. |
| **Total Control (TC) No.1 (Weeks 1 – 7 )** | 2 | |
| **Module 4. Polynomial Functions and Interpolations** | | | | |
| 8  -  9  -10 | **Lecture 15-20.** Properties of Polynomials. Lagrange Interpolation. Newton Interpolation. Gauss Interpolation. Stirling and Bessel Interpolations. Errors of Polynomial Interpolation. Polynomial Approximation.  **Seminar 8-10.** Computer Modellingof Lagrange, Newton, Gauss Interpolations. | 6  3 | | **SSI-8-10**  Properties of Polynomials.  Stirling and Bessel Interpolations. Polynomial Approximation Problems. |
| **Module 5. Differential and Integral Calculus** | | | | |
| 11-12  -  13  -  14  -  15 | **Lecture 21-30.** Derivative and Finite Differences. Higher-Order Numerical Derivatives. Multipoint First-Order Numerical Derivatives. Richardson Extrapolation. Integrals and Finite Sums. Newton-Cotes Integration Rules. Trapezium formula. Simpson formula. Gaussian Quadrature Rules. **Estimation of the Integral by Monte Carlo Methods.** Random Variable. Discrete Random Variable. Probability Density and Distribution Functions (PDF). Expectation (Mean), Variance and Transforms. Random Variables Modelling, Neumann Modelling. Chebyshev Inequality. Essential Random Sample Methods.  **Seminar 11-15.** Computer Modellingof theIntegral by Trapezium, Simpson and Monte Carlo methods. | 10  5 | | **SSI-11-15**  First-order and second-order forward and backward numerical derivatives for concrete functions. First-order and second-order central numerical derivatives for concrete functions. Variance Sample. Computer Modelling of Random Variables. |
|  | **Total Control (TC) No.2 (Weeks 8 – 15 )** | 2 | |  |

**References**

**Basic:**

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2. Matheus Grasselli, Dmitry Pelinovsky. Numerical Mathematics. Narosa Publishing House. India. 2009.

**Additional:**

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2. Митчелл Э., Уэйт Р.. Метод конечных элементов для уравнений с частными производными. М., Мир, 1981.
3. Трауб Дж.. Итерационные методы решения уравнений. М., Мир, 1989.
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частных производных. М., ИЛ, 1963.

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9. Березин И.С., Жидков Н.П. Методы вычислений. М., Наука. 1966. Том 1. 632 с. Том 2. 635 c.

10. Михлин С.Г. Численная реализация вариационных методов. М., Наука. 432 с.

11. Демидович Б.П., Марон И.А. Основы вычислительной математики. М., Наука. 1970. 664 с.

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13. Милн В.Э. Численный анализ. М., ИЛ. 1951. 291 с.

14. Воеводин В.В. Вычислительные основы линейной алгебры. М.., Наука. 1977. 303 с.

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15. Годунов С.К., Рябенький В.С. Разностные схемы. М., Наука. 1977. 439 с.

17. Марчук Г.И., Лебедев В.И. Численные методы в теории переноса нейтронов. М.,

Атомиздат. 1981. 456 с.

18. Демидович Б.П., Марон И.А., Шувалова Э.З. Численные методы анализа. М., Наука. 1967.

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19. Кунц К.С. Численный анализ. Киев, «Техника». 1964. 390 с.

20. Самарский А.А., Николаев Е.С. Методы решения сеточных уравнений. М., Наука. 1978.

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21. Годунов С.К. Решение систем линейных уравнений. Новосибирск. Наука. 1980. 177 с.

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университета. 1988. 333 с.

**Control Test: twice.**

**SSI***:* **a few times.**

**Criterion of grade of the knowledge, marks in percent**

|  |  |
| --- | --- |
| *Lecture* | *30* |
| *SSI – theory* |
| *SSI – practice(seminar)* | *15* |
| *Total written examination* | *45* |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Weeks** | **Lecture** | | **Seminar** | | **SSI** | | **TC** | **TOTAL** |
|  | **No.** | **Mark** | **No.** | **Mark** | **No.** | **Mark** |  | **Mark** |
| 1 | 1 | 0,5 | 1 | 2 | 1 | 0,5 |  | 3 |
| 2 | 2 | 0,5 | 2 | 2 | 2 | 0,5 |  | 3 |
| 3 | 3 | 0,5 | 3 | 2 | 3 | 0,5 |  | 4 |
| 4 | 4 | 0,5 | 4 | 4 | 4 | 0,5 |  | 5 |
| 5 | 5 | 0,5 | 5 | 5 | 5 | 0,5 |  | 6 |
| 6 | 6 | 0,5 | 6 | 3 | 6 | 0,5 |  | 4 |
| 7 | 7 | **-** | 7 | 3 | 7 | **-** | 3 | 6 |
| **Total: Weeks 1-7** |  | **3** |  | **21** |  | **3** | **3** | **30** |
| 8 | 8 | 0,5 | 8 | 2 | 8 | 0,5 |  | 3 |
| 9 | 9 | 0,5 | 9 | 4 | 9 | 0,5 |  | 5 |
| 10 | 10 | 0,5 | 10 | 3 | 10 | 0,5 |  | 4 |
| 11 | 11 | 0,5 | 11 | 3 | 11 | 0,5 |  | 4 |
| 12 | 12 | 0,5 | 12 | 2 | 12 | 0,5 |  | 3 |
| 13 | 13 | 0,5 | 12 | 1 | 13 | 0,5 |  | 2 |
| 14 | 14 | - | 12 | 3 | 14 | - |  | 3 |
| 15 | 15 | - | 12 | 3 | 15 | - | 3 | 6 |
| **Total: Weeks 8-15** |  | **3** |  | **21** |  | **3** | **3** | **30** |
| **Total: Weeks 1-15** |  | **6** |  | **42** |  | **6** | **6** | **60** |

**The scale of mark of knowledge:**

|  |  |  |  |
| --- | --- | --- | --- |
| **Letter symbol of mark** | **Digital of mark (GPA)** | **Mark (percent)** | **Mark on tradition system** |
| A | 4 | 95-100 | “excellent” |
| A- | 3,67 | 90-94 |
| B+ | 3,33 | 85-89 | “good” |
| B | 3 | 80-84 |
| B- | 2,67 | 75-79 |
| C+ | 2,33 | 70-74 | “satisfactory” |
| C | 2 | 65-69 |
| C- | 1,67 | 60-64 |
| D+ | 1,33 | 55-59 |
| D | 1 | 50-54 |
| F | - | 0-49 | “unsatisfactory” |
| I | - | - | “Incomplete discipline” |
| W | - | - | “Renunciation of discipline” |
| AW | - | - | “Deduction off discipline” |
| AU | - | - | “To take a discipline” |
| P/NP (Pass / No Pass) | - | 65-100/0-64 | “Pass / No Pass” |

Sitting of the chair consideration

Protocol No. , , 2012

**Acting as chief of chair M&K M,**

PhD, docent \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ D. Zhakebaev

**Lecturer**

Doctor of Science, professor \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Кanat Shakenov